

I-brane dynamics

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ABSTRACT: We study the dynamics near a $1+1$ dimensional intersection of two orthogonal stacks of fivebranes in type IIB string theory, using an open string description valid at weak coupling, and a closed string description valid at strong coupling. The weak coupling description suggests that this system is invariant under eight supercharges with a particular chirality in $1+1$ dimensions, and its spectrum contains chiral fermions localized at the intersection. The closed string description leads to a rather different picture — a three dimensional Poincare invariant theory with a gap and sixteen supercharges. We show that this dramatic change in the behavior of the system is partly due to anomaly inflow. Taking it into account leads to a coherent picture, both when the fivebranes in each stack are coincident and when they are separated.

KEYWORDS: Brane Dynamics in Gauge Theories, AdS-CFT Correspondence, Gauge-gravity correspondence.

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1. Introduction and summary

In this paper we study a system consisting of two stacks of fivebranes in type IIB string theory, which intersect on an $\mathbb{R}^{1,1}$. This system can be analyzed from a number of different points of view and exhibits some interesting features:

- (1) Holography: one might expect that the dynamics at the 1+1 dimensional intersection of the two sets of fivebranes should be holographically related to a 2 + 1 dimensional bulk theory, with the extra dimension being the radial direction away from the intersection. In fact the bulk description includes *two* radial directions away from each set of fivebranes, and is 3 + 1 dimensional. The corresponding boundary theory is 2 + 1 dimensional.
- (2) Near-horizon symmetry enhancement: inspection of the brane configuration suggests that the theory at the intersection of the fivebranes should be invariant under 1 + 1 dimensional Poincare symmetry, $ISO(1, 1)$, and eight supercharges with a particular chirality in 1 + 1 dimensions. However, the near-horizon geometry describes a 2 + 1 dimensional theory with Poincare symmetry $ISO(2, 1)$, and twice as many supercharges as one would naively expect – eight copies of the two dimensional spinor of $Spin(2, 1)$. Interestingly, the superalgebra is not the standard $\mathcal{N} = 8$ supersymmetry in three dimensions.¹ This symmetry enhancement is surprising since it implies that there are no normalizable states localized at the intersection, while the weakly coupled open string analysis leads to a large number of such states. In particular, the intersection is expected to carry chiral degrees of freedom in 1 + 1 dimensions, which cannot be lifted, and whose presence is inconsistent with the enhanced (super-) symmetry of the near-horizon geometry.
- (3) Anomaly inflow: the chiral modes living at the intersection of the fivebranes carry non-zero anomalies under the gauge symmetries on the fivebranes. These anomalies are cancelled by inflow from the bulk of the fivebranes [2]. The fact that the anomaly inflow mechanism is at play here is the first sign that the dynamics at the intersection is not decoupled from that in the bulk of the fivebranes. As we will see, the higher dimensional perspective is essential in understanding various features of this system.

1.1 The fivebrane system

A concrete realization of our brane system is the following. Consider k_1 $D5$ -branes stretched in the directions (012345), and k_2 $D5$ -branes stretched in (016789). This configuration is invariant under $ISO(1, 1) \times SO(4)_{2345} \times SO(4)_{6789}$. It also preserves eight supercharges, which satisfy the constraints

$$\begin{aligned} \epsilon_L &= \Gamma^{012345} \epsilon_R, \\ \epsilon_L &= \Gamma^{016789} \epsilon_R . \end{aligned} \tag{1.1}$$

Taking into account the fact that the spinors ϵ_L, ϵ_R in (1.1) have the same chirality in 9 + 1 dimensions, one finds that the solutions of (1.1) are chiral in $\mathbb{R}^{1,1}$.

At long distances, one can describe the dynamics of this brane configuration by a low energy effective field theory. In addition to the familiar six dimensional $U(k_i)$ gauge theory with sixteen supercharges on each set of $D5$ -branes, the low lying spectrum contains $k_1 k_2$

¹This point was independently noticed in [1].

complex, chiral (Weyl) fermions, which transform in the representation $(\mathbf{k}_1, \bar{\mathbf{k}}_2)$ of the two gauge groups. These fermions originate from fundamental strings stretched between the two stacks of fivebranes. We will take them to be left-moving below. Their presence breaks the left-moving² spacetime SUSY; the eight right-moving supercharges remain unbroken.

The fermions have central charge $c_L = k_1 k_2$, $c_R = 0$, and thus carry a non-zero gravitational anomaly in $1 + 1$ dimensions. As we change various continuous parameters defining the brane configuration, such as the string coupling and the separations between different parallel fivebranes in a given stack, $c_L - c_R = k_1 k_2$ cannot change.

The fermions are also anomalous under $SU(k_1) \times SU(k_2) \times U(1)$. To exhibit the anomaly, it is convenient to use the fact that we can replace them by the holomorphic current algebra

$$SU(k_1)_{k_2} \times SU(k_2)_{k_1} \times U(1) . \tag{1.2}$$

Hence, the $SU(k_1)$ gauge group on the first set of fivebranes sees an anomaly proportional to k_2 localized on a codimension four defect, and vice-versa. There is also a $U(1)$ anomaly coming from the last factor in (1.2).

1.2 Weak coupling analysis

To study the dynamics of the fermions, it is important to include in the effective action the terms that couple them to the six dimensional gauge fields on the fivebranes and cancel the anomalies. As we will see in sections 2 and 3, these have an interesting dynamical effect. Naively, the fermions are localized at the intersection point of the two sets of fivebranes. In fact, we will see that they are supported away from the intersection. The amount by which they are displaced is governed by the six dimensional gauge couplings of the $SU(k_i)$ gauge theories, $g_i^2 = g_s l_s^2$. We will see that each of the three factors in (1.2) is displaced in a different way in the transverse space.

The displacements, which are proportional to the gauge couplings g_i , vanish in the weak coupling limit $g_s \rightarrow 0$. In this limit, the picture becomes indistinguishable from what one finds by studying open strings stretched between $D5$ -branes. This is not surprising, since in this limit all couplings, including those between the modes at the intersection and those in the bulk of the fivebranes, vanish. On the other hand, as g_s increases, the effect of the anomaly inflow becomes more pronounced, and the picture deviates more from the weakly coupled open string one.

Let us describe the low energy field theory in more detail. Here we discuss the case when all fivebranes in a given stack are coincident. The case of separated fivebranes will be studied below. We parameterize the $1 + 1$ dimensional intersection of the two stacks of branes by (x^0, x^1) . The other eight coordinates can be organized into two \mathbb{R}^4 's which we will parameterize by spherical coordinates. We will denote by u the radial direction along the first set of branes (it is the radial direction of the \mathbb{R}^4 transverse to the second set of branes); v is the radial direction along the second set of branes (and the radial direction of the space transverse to the first set of branes). The intersection is at $u = v = 0$.

²This should not be confused with the notion of left and right in equation (1.1), which refers to worldsheet chirality.

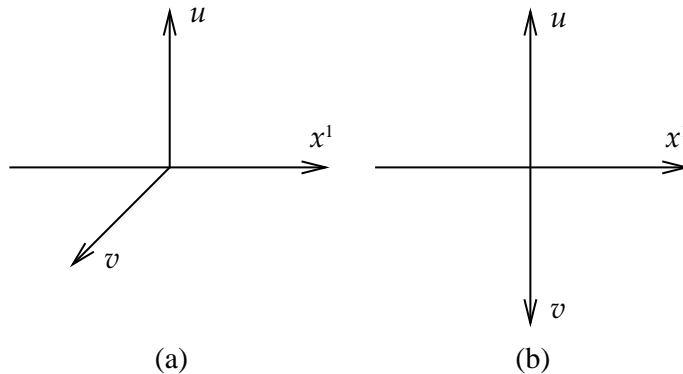


Figure 1: The spatial part of the $2+1$ dimensional space we consider. One spatial direction is x^1 and the other is constructed out of u and v . Figure 1a stresses that in the embedding space, the u and v directions are orthogonal. Figure 1b stresses that u and v can be combined into a second spatial direction.

We will find it convenient to reduce the system on the two S^3 's of these spherical coordinates and study the resulting effective three dimensional theory. In addition to the two coordinates (x^0, x^1) the base space has another spatial direction made out of the half-lines³ labeled by u and v , glued at $u = v = 0$ (see figure 1).

Thus we have a $2+1$ dimensional system with a “domain wall” located on the real axis $u = v = 0$. The coordinates along the wall are (x^0, x^1) . For zero coupling the free fermions and their current algebra (1.2) are supported on the “domain wall.” In addition to these fermions the system also has gauge fields which propagate in the entire $2+1$ dimensional space. On the upper half plane we have an $SU(k_1)$ gauge theory, while on the lower one we have an $SU(k_2)$ gauge theory. There is also a $U(1)$ gauge field which propagates on the whole plane. These theories have Chern-Simons terms which are due to the fact that we are studying fivebranes wrapped on three-spheres with non-vanishing three-form flux. Because of these Chern-Simons terms the gauge fields are massive.

In figure 2 we describe the location of the chiral modes in our $2+1$ dimensional system. Here we describe only the nontrivial directions u and v and omit the $\mathbb{R}^{1,1}$ labeled by (x^0, x^1) . In order to stress that u and v originate from different branes, and are orthogonal in the original $\mathbb{R}^{9,1}$, the u axis is drawn orthogonal to the v axis, as in fig. 1(a). For zero coupling the fermions are supported at $u = v = 0$ (see figure 2a). However, as we will see in sections 2 and 3, because of the interaction with the gauge fields, when the gauge coupling constants are nonzero, the current algebra is displaced. Different parts of (1.2) “move” to different locations (see figure 2b). $SU(k_1)_{k_2}$ is supported at $u \sim \sqrt{g_1^2 k_2}$, $SU(k_2)_{k_1}$ is supported at $v \sim \sqrt{g_2^2 k_1}$ and the $U(1)$ factor is supported at both these points.

As the coupling constant is increased, the current algebra moves to infinity and the low energy theory around the real axis in figure 1 becomes very simple. First, the system has a gap there. Second, the topological degrees of freedom associated with the Chern-Simons theories in the lower and upper half planes combine nicely. We have $SU(k_1)$ Chern-Simons

³Recall that these are radial directions, therefore $u, v \geq 0$.

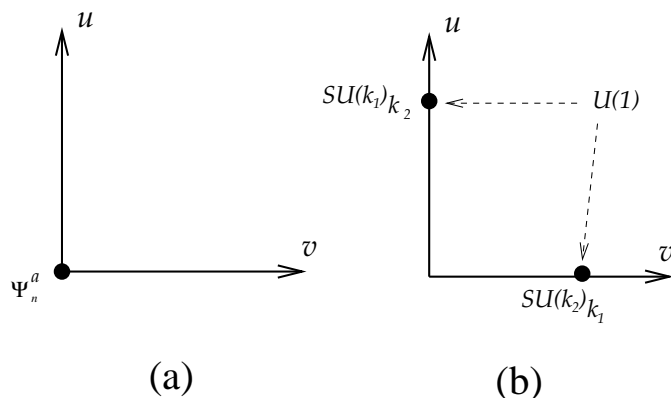


Figure 2: (a) At zero coupling all the fermions are supported at the intersection. (b) At finite coupling they move away from the intersection. Different parts of the current algebra move in different directions. $SU(k_1)_{k_2}$ is supported at $u \sim \sqrt{g_1^2 k_2}$; $SU(k_2)_{k_1}$ is supported at $v \sim \sqrt{g_2^2 k_1}$; the $U(1)$ mode is supported at both of these points.

theory with level k_2 in the upper half plane and $SU(k_2)$ level k_1 in the lower half plane. Due to level-rank duality, these two theories are the same. Therefore, the low energy dynamics of this brane configuration is a Chern-Simons theory with gauge group $SU(k_1)_{k_2} \times U(1)$ both in the lower and in the upper half plane. The “domain wall” along the real axis disappears and our low energy theory is fully invariant under 2 + 1 dimensional Poincare symmetry.

The existence of this 2 + 1 dimensional Poincare symmetry has immediate implications to the supersymmetry of the system. The eight supercharges (1.1) do not fit into a representation of this symmetry. As the Poincare symmetry is enhanced from two dimensions to three, the supersymmetry is also enhanced from eight to sixteen supercharges. The superalgebra is not the standard one – the anticommutator of two supercharges includes in addition to the momentum operator also the generators of the $SO(4) \times SO(4)$ R-symmetry of the problem.⁴

1.3 Strong coupling analysis

In section 4 we turn to the strong coupling limit of our system. Here it is convenient to perform an S-duality transformation to a system of $NS5$ -branes.⁵ When all the fivebranes

⁴It is not known how to write a three dimensional lagrangian for gauge fields with Chern-Simons terms and this much supersymmetry (for a related recent discussion see [3]). In the past, the search for such lagrangians was based on the standard extended supersymmetry algebra. The string theory problem we are studying suggests that with the unconventional superalgebra that we find, lagrangians with sixteen supercharges and Chern-Simons terms can be written. It would be interesting to see whether kinetic terms for the gauge fields can be included in such lagrangians. This is not entirely clear from the string theory perspective, since at the scale of the gauge coupling our theory becomes higher dimensional, but we believe that it is nevertheless possible.

⁵The discussion can be extended to any other kind of (p, q) fivebranes. The $SL(2, Z)$ symmetry of IIB string theory implies that all such configurations have the same amount of supersymmetry, low lying spectrum and dynamics. In particular, the chiral fermions, the gauge fields and the action that couples the

are coincident, the near-horizon geometry is

$$\mathbb{R}^{2,1} \times \mathbb{R}_\phi \times SU(2)_{k_1} \times SU(2)_{k_2}. \tag{1.3}$$

Here, \mathbb{R}_ϕ is one combination of the radial directions away from the two sets of fivebranes, and the coordinates of $\mathbb{R}^{2,1}$ are x^0, x^1 and another combination of the two radial directions. The two $SU(2)$'s describe the angular three-spheres corresponding to $(\mathbb{R}^4)_{2345}$ and $(\mathbb{R}^4)_{6789}$. The fact that (1.3) is an exact solution of the classical string theory equations of motion allows us to obtain information about the intersecting fivebrane system, which is not accessible via a gauge theory analysis.

As mentioned above, the geometry (1.3) has the interesting property that it exhibits a higher symmetry than the full brane configuration. In particular, the combination of radial directions away from the intersection that enters $\mathbb{R}^{2,1}$ appears symmetrically with the other spatial direction, and the background has a higher Poincare symmetry, $ISO(2, 1)$, than the expected $ISO(1, 1)$. It also has twice as much supersymmetry.

A natural question is whether this higher symmetry is an exact property of string theory in the background (1.3), or whether it is broken by quantum effects. In weakly coupled string theory, classical symmetries are usually symmetries of the full theory, but the theory on (1.3) is not weakly coupled. As $\phi \rightarrow -\infty$, the string coupling becomes strong and one could imagine that in that region the symmetry of (1.3) is broken to a smaller one.

A similar divergence appears in the six dimensional background corresponding to a single stack of k parallel fivebranes in type II string theory. In that case, the region $\phi \rightarrow -\infty$ describes the long distance behavior of the theory on the branes [4], which has an alternative description as a six dimensional $U(k)$ SYM theory with sixteen supercharges for type IIB string theory, and as the (2, 0) SCFT for IIA. In both cases, the low energy theory has non-Abelian degrees of freedom which from the point of view of the geometry are D-branes living near the singularity. The fact that they are light implies that string perturbation theory must break down there.

It is natural to follow the same logic in our case. The background (1.3) becomes strongly coupled near the intersection of the branes partly due to the fact that the low energy dynamics near the intersection involves a non-Abelian gauge theory. Thus, to see whether the $2 + 1$ dimensional super-Poincare symmetry is broken in the strong coupling region, one needs to analyze the gauge theory.

The results of this analysis are described in the previous subsection (and in more detail in sections 2, 3). The gauge theory dynamics displaces the chiral fermions, which naively live at the intersection, by an amount of order g_{YM} , which for $NS5$ -branes is l_s . Since the near-horizon limit that gives rise to (1.3) focuses on distances much smaller than l_s , the chiral fermions are not part of the theory in the near-horizon limit. In fact, in taking the near-horizon limit, each of the three factors in (1.2) is sent to infinity in a different

two are the same for all (p, q) . The only part of the gauge theory analysis that depends on (p, q) is the value of the gauge coupling on the fivebranes. Since the gauge coupling determines the extent by which the holomorphic currents (1.2) are displaced from the intersection of the branes, the latter depends on the type of fivebranes that are used in the construction.

direction. The two $SU(k_i)$ are sent to plus and minus infinity in one of the two spatial directions of $\mathbb{R}^{2,1}$, while the $U(1)$ is sent to infinity in the weak coupling direction of \mathbb{R}_ϕ .

So far we restricted attention to the extreme infrared behavior of the intersecting five-brane system, and found that it is consistent with Poincare symmetry in $2 + 1$ dimensions. It is natural to postulate that a much stronger statement, that we did not prove, is true: the near-horizon limit eliminates *all* $1+1$ dimensional degrees of freedom, and leaves behind a $2 + 1$ dimensional theory with a larger symmetry. We present some evidence for this in section 6.

1.4 The big picture

The discussion above focused on the dynamics associated with the intersection in the case when the fivebranes are coincident. Some new features appear when we separate the two sets of fivebranes in the corresponding transverse \mathbb{R}^4 's. One consequence of this separation is a breaking of the Poincare symmetry of the near-horizon geometry back to $ISO(1,1)$, and of half of the sixteen supercharges, back to the original eight. The $U(k_i)$ gauge groups are generically broken to $U(1)^{k_i}$ and an additional dimensionless parameter, the mass of the W-bosons of the broken gauge theory in units of the inverse gauge coupling (M_s for $NS5$ -branes), appears.

For $M_W \ll M_s$, the deformed near-horizon geometry which generalizes (1.3) is still strongly coupled, and the useful description of the system is given by the gauge theory described earlier. On the other hand, for $M_W \gg M_s$, the gauge theory description is in general not valid⁶, and the useful description is in terms of the near-horizon geometry, a deformation of (1.3), which never develops large string coupling. By analyzing the bulk geometry we show that all $k_1 k_2$ chiral modes are present, as certain chiral modes of RR fields in the geometry, and find their locations in the transverse space.

In section 5 we combine the information obtained from the gauge theory analysis for $M_W \ll M_s$, with that obtained from the near-horizon geometry of the fivebranes for $M_W \gg M_s$ to reconstruct the behavior of the chiral modes as we vary the different parameters. In section 6 we point out a few extensions of our work and directions for future study. Some of our worldsheet computations are presented in the appendix.

2. Gauge theory analysis for $k_1 = k_2 = 1$

In this section we will study the low energy field theory corresponding to the intersection of two $D5$ -branes, one stretched in the directions (012345), the other in (016789). As discussed in the introduction, this system contains a single complex chiral fermion, Ψ , that lives at the intersection. This fermion carries equal and opposite charges under the $U(1)$ gauge fields living on the two intersecting fivebranes. We will describe the combined system of the fermion and the gauge fields, and in particular analyze the effects of anomaly inflow on the dynamics.

⁶Although, as in [5], it may still be useful for some purposes.

2.1 The lagrangian

It is useful to parameterize the ten dimensional spacetime as follows:

$$ds^2 = 2dx^+ dx^- + du^2 + u^2 d\Omega_u^2 + dv^2 + v^2 d\Omega_v^2. \quad (2.1)$$

Here, (u, Ω_u) are spherical coordinates on $(\mathbb{R}^4)_{2345}$ and (v, Ω_v) are spherical coordinates on $(\mathbb{R}^4)_{6789}$. The two intersecting fivebranes share the directions $x^\pm = (x^0 \pm x^1)/\sqrt{2}$; in addition, one is wrapped on (u, Ω_u) , while the other is wrapped on (v, Ω_v) . The two fivebranes intersect at $u = v = 0$.

The low energy gauge theory includes a $U(1)$ gauge field that lives on the worldvolume of each kind of fivebrane, which we will refer to as $A^{(1)}$ and $A^{(2)}$, respectively. $A^{(1)}$ lives in the six dimensional spacetime labeled by (x^\pm, u, Ω_u) , while $A^{(2)}$ is a function of (x^\pm, v, Ω_v) . We can expand each of the gauge fields in harmonics on the corresponding three-sphere. Since we are only interested in the lowest lying states living near the intersection, we integrate out the higher harmonics and keep in the effective action only the s-waves of $A^{(i)}$ on the spheres.

We expect the low energy effective action near the intersection to be a sum of three terms: a 2 + 1 dimensional action for the field $A^{(1)}(x^\pm, u)$, the s-wave of $A^{(1)}$ on the sphere labeled by Ω_u ; a similar 2 + 1 dimensional action for the s-wave of $A^{(2)}$, $A^{(2)}(x^\pm, v)$; and a 1 + 1 dimensional action which lives at the intersection $u = v = 0$. In the rest of this subsection we will write this action explicitly.

We start with the 1 + 1 dimensional action associated with the intersection. The complex fermion Ψ couples to the gauge field $A^{(1)} - A^{(2)}$ at the intersection point, $u = v = 0$, which we will denote by $A^{(1)}(0) - A^{(2)}(0)$, suppressing the dependence on x^\pm . As is standard in two dimensional field theory, one can integrate out the fermion and represent its dynamics in terms of its contribution to the effective action of the gauge field,⁷

$$\begin{aligned} \mathcal{L}_{\text{ferm}} = & \left(A_+^{(1)}(0) - A_+^{(2)}(0) \right) \frac{\partial_-}{\partial_+} \left(A_+^{(1)}(0) - A_+^{(2)}(0) \right) - \\ & - \left(A_+^{(1)}(0) - A_+^{(2)}(0) \right) \left(A_-^{(1)}(0) - A_-^{(2)}(0) \right). \end{aligned} \quad (2.2)$$

This effective lagrangian is anomalous. Under the gauge transformation

$$\delta A_\mu^{(i)} = \partial_\mu \epsilon^{(i)}, \quad i = 1, 2, \quad (2.3)$$

$\mathcal{L}_{\text{ferm}}$ transforms as follows

$$\delta \mathcal{L}_{\text{ferm}} = \left(\epsilon^{(1)}(0) - \epsilon^{(2)}(0) \right) \left(F_{+-}^{(1)}(0) - F_{+-}^{(2)}(0) \right). \quad (2.4)$$

As explained in [2], this anomaly is cancelled by inflow from the bulk of the two fivebranes, via local terms in the lagrangians on the two branes.

⁷The overall coefficient of the two dimensional lagrangian implies a choice of normalization of the charge of the fermion under the $U(1)$ gauge fields $A^{(i)}$, or equivalently a choice of normalization of the gauge fields themselves. This determines the coefficients of all the other terms that appear below.

There are two relevant types of terms. One involves the coupling of the Chern-Simons form of the gauge field $A^{(i)}$ to the RR three-form field strength, F_3 . For example, on brane 1, this takes the form⁸

$$\int F_3 \wedge A^{(1)} \wedge F^{(1)}. \tag{2.5}$$

The three-sphere labeled by Ω_u on which fivebrane 1 is wrapped is transverse to fivebrane 2. Therefore, there is one unit of F_3 flux going through it. Thus after integrating over the sphere, we find a three dimensional Chern-Simons action for the s-wave of $A^{(1)}$, $A^{(1)}(x^\pm, u)$, and a similar one for $A^{(2)}(x^\pm, v)$:

$$\begin{aligned} \mathcal{L}_{\text{CS}} = & \int_0^\infty du \left(A_+^{(1)} F_{-u}^{(1)} + A_-^{(1)} F_{u+}^{(1)} + A_u^{(1)} F_{+-}^{(1)} \right) + \\ & + \int_0^\infty dv \left(A_+^{(2)} F_{-v}^{(2)} + A_-^{(2)} F_{v+}^{(2)} + A_v^{(2)} F_{+-}^{(2)} \right). \end{aligned} \tag{2.6}$$

As is standard in Chern-Simons theory with a boundary, the lagrangian (2.6) is not gauge invariant. Under the gauge transformation (2.3) it transforms as follows:

$$\delta \mathcal{L}_{\text{CS}} = -\epsilon^{(1)}(0) F_{+-}^{(1)}(0) - \epsilon^{(2)}(0) F_{+-}^{(2)}(0), \tag{2.7}$$

where we used the fact that we only require invariance under transformations that go to zero at infinity, *i.e.* the gauge parameters satisfy $\epsilon^{(i)}(\infty) = 0$.

The second type of Chern-Simons term that is relevant for anomaly inflow involves a coupling of the worldvolume gauge fields to the self-dual RR field strength, F_5 ,

$$\int F_5 \wedge A^{(i)}. \tag{2.8}$$

Since the D5-branes do not couple to F_5 , naively this term is irrelevant for our discussion. However when the gauge fields on both branes are turned on, this term has a non-trivial effect. For example, if we turn on a non-zero gauge field on brane 2, with field strength $F_{+-}^{(2)}$, this induces a non-zero F_5 on brane 1 of the form

$$F_5 = F_3 \wedge F^{(2)}(0). \tag{2.9}$$

Here, F_3 is the RR three-form field strength induced by brane 2 on the sphere Ω_u , as in (2.5), and $F^{(2)}$ is evaluated at $v = 0$, since this is the location of brane 1. Substituting (2.9) into (2.8), reducing to s-waves, and integrating over the spheres, one finds the following contribution to the action:

$$\mathcal{L}_{\text{mix}} = -F_{+-}^{(2)}(0) \int_0^\infty du A_u^{(1)} - F_{+-}^{(1)}(0) \int_0^\infty dv A_v^{(2)}. \tag{2.10}$$

The total anomaly inflow lagrangian is given by

$$\mathcal{L}_{\text{inflow}} = \mathcal{L}_{\text{CS}} + \mathcal{L}_{\text{mix}}. \tag{2.11}$$

⁸We are not careful with the overall normalization here; we will determine it in our conventions momentarily, after reducing to 2 + 1 dimensions.

The full action,

$$S_0 = \int dx^+ dx^- (\mathcal{L}_{\text{ferm}} + \mathcal{L}_{\text{inflow}}), \quad (2.12)$$

is anomaly free.⁹

In addition to the fermion contribution and anomaly inflow lagrangian, we also need to keep the kinetic terms of $A^{(1)}$ and $A^{(2)}$. After reducing on the spheres, these take the form

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & \frac{1}{g_1^2} \int_0^\infty du u^3 \left[\frac{1}{2} (F_{+-}^{(1)})^2 - F_{u+}^{(1)} F_{u-}^{(1)} \right] + \\ & + \frac{1}{g_2^2} \int_0^\infty dv v^3 \left[\frac{1}{2} (F_{+-}^{(2)})^2 - F_{v+}^{(2)} F_{v-}^{(2)} \right]. \end{aligned} \quad (2.13)$$

Here g_1, g_2 are the six dimensional gauge couplings, $g_i^2 \simeq g_s l_s^2$ for $D5$ -branes, or $g_i^2 \simeq l_s^2$ for $NS5$ -branes, and we have absorbed a numerical factor in the definition of g_i . In principle, one should also add higher derivative terms to (2.12), but we expect them not to alter the picture obtained below.

2.2 Equations of motion and solutions

The full effective action

$$S_{\text{full}} = \int dx^+ dx^- (\mathcal{L}_{\text{ferm}} + \mathcal{L}_{\text{inflow}} + \mathcal{L}_{\text{kin}}) \quad (2.14)$$

is quadratic in the gauge fields; therefore, to solve the theory it is enough to study solutions to the equations of motion of (2.14). In this subsection we will write down these equations and find the solution corresponding to the chiral mode associated with the fermion Ψ .

Due to the presence of the boundary at $u = v = 0$, we have to vary separately with respect to the gauge field in the bulk and on the boundary. Varying with respect to $A_+^{(1)}$ in the bulk (*i.e.* for generic u) we find

$$\frac{1}{g_1^2} \partial_u (u^3 F_{u-}^{(1)}) - 2F_{u-}^{(1)} + \frac{u^3}{g_1^2} \partial_- F_{+-}^{(1)} = 0. \quad (2.15)$$

Varying with respect to $A_-^{(1)}$ gives

$$\frac{1}{g_1^2} \partial_u (u^3 F_{u+}^{(1)}) + 2F_{u+}^{(1)} + \frac{u^3}{g_1^2} \partial_+ F_{-+}^{(1)} = 0, \quad (2.16)$$

and the variation of $A_u^{(1)}$ gives

$$2F_{+-}^{(1)} - F_{+-}^{(2)}(0) - \frac{u^3}{g_1^2} (\partial_- F_{u+}^{(1)} + \partial_+ F_{u-}^{(1)}) = 0. \quad (2.17)$$

Similar equations are obtained with $1 \leftrightarrow 2$, $u \leftrightarrow v$.

⁹Here we restrict to gauge anomalies, and ignore gravitational ones.

Due to the presence of the Chern-Simons term we expect the equations of motion (2.15) – (2.17) to describe a massive particle in three dimensions with a mass that depends on u (because of the factor of u^3 in \mathcal{L}_{kin}). Indeed, differentiating equation (2.15) w.r.t. x^+ , and equation (2.16) w.r.t. x^- , and subtracting the two we find, using (2.17)

$$\partial_u \left(u^3 \partial_u F_{+-}^{(1)} \right) + 2u^3 \partial_+ \partial_- F_{+-}^{(1)} - \frac{2g_1^4}{u^3} \left(2F_{+-}^{(1)} - F_{+-}^{(2)}(0) \right) = 0. \quad (2.18)$$

The mass of the particle goes to zero as $u \rightarrow \infty$, but it does not give rise to massless degrees of freedom in the $\mathbb{R}^{1,1}$ labeled by x^\pm . One can show that equation (2.18) does not have solutions with $\partial_+ \partial_- F_{+-}^{(1)} = 0$ that are regular as $u \rightarrow 0$ and go to zero as $u \rightarrow \infty$. The massless regular solutions of (2.18) go to a non-zero constant at $u = \infty$ and correspond to a constant electric field in $\mathbb{R}^{1,1}$. In other words, (2.18) describes a massive degree of freedom in the bulk of fivebrane 1.

Thus, we set

$$F_{+-}^{(1)} = F_{+-}^{(2)} = 0. \quad (2.19)$$

Equations (2.15) and (2.16) can then be solved for $F_{u\pm}^{(1)}$

$$F_{u\pm}^{(1)} = \frac{h_\pm}{u^3} e^{\pm \frac{g_1^2}{u^2}}, \quad (2.20)$$

where h_\pm depend on x^\pm but not on u . While the solution for $F_{u-}^{(1)}$ is well behaved as $u \rightarrow 0, \infty$, the solution for $F_{u+}^{(1)}$ is highly divergent as $u \rightarrow 0$, so we discard it, and set $h_+ = 0$. Moreover, substituting in (2.17) we see that h satisfies the constraint

$$\partial_+ h_- = 0. \quad (2.21)$$

Thus, it is a chiral degree of freedom, $h_- = h_-(x^-)$.

Next we consider the equations of motion of the boundary variables. Varying w.r.t. $A_+^{(1)}(0)$ we get

$$\int_0^\infty dv F_{-v}^{(2)} = \frac{2}{\partial_+} \left(F_{-+}^{(1)}(0) - F_{-+}^{(2)}(0) \right) \equiv g(x^-). \quad (2.22)$$

In deriving this equation we used the fact that $\lim_{u \rightarrow 0} u^3 F_{u-}^{(1)} = 0$, (2.20). Note also that the quantity $g(x^-)$ defined by (2.22) is indeed chiral, $\partial_+ g = 0$, due to (2.19). Again, there are two more equations obtained by exchanging $1 \leftrightarrow 2$.

To recapitulate, we see that including the interaction of the chiral fermion Ψ with the gauge fields living on the two fivebranes via anomaly inflow displaces it from the intersection of the two branes at $u = v = 0$ by an amount of order the gauge coupling on the fivebranes. One finds a single chiral degree of freedom, which can be thought of as $h_-(x^-)$ (2.20), that lives at $u \sim g_1$. Equation (2.22) relates it to the current located at the origin. The equations of motion on the second brane relate this mode to a solution similar to (2.20) that lives at $v \sim g_2$. The situation is depicted in figure 2b.

At short distances from the intersection, $u, v \ll g_i$, the fermions are absent and the physics is described by $U(1)$ Chern-Simons theory on $\mathbb{R}^{2,1}$.

3. Gauge theory analysis for general k_1, k_2

A natural generalization of the brane configuration discussed in the previous section involves k_1 fivebranes stretched in (012345), intersecting k_2 fivebranes stretched in (016789). At the intersection we have $k_1 k_2$ complex fermions Ψ_n^a ; $n = 1, \dots, k_1$; $a = 1, \dots, k_2$. The fermions are coupled to $U(k_1) \times U(k_2)$ gauge fields. As mentioned in the introduction, one can construct out of the fermions left-moving currents of the affine Lie algebra $SU(k_1)_{k_2} \times SU(k_2)_{k_1} \times U(1)$. As a check, the Sugawara central charge of the current algebra agrees with that of the fermions:

$$\frac{k_2(k_1^2 - 1)}{k_1 + k_2} + \frac{k_1(k_2^2 - 1)}{k_1 + k_2} + 1 = k_1 k_2 . \tag{3.1}$$

In generalizing the discussion of the previous section to this case there are two new issues to consider. One is that the gauge theory in question is now non-Abelian, so the dynamics is more complicated. The other is that the brane configuration admits deformations that correspond to separating each stack of fivebranes in the transverse \mathbb{R}^4 , and one may ask how the low energy behavior of the theory depends on the separations.

In this section we will examine these issues in turn. We start with a discussion of the dynamics for the case of coincident fivebranes, and then move on to the separated case.

3.1 Coincident fivebranes

Like in the Abelian case, we integrate out the fermions, and replace them by their contribution to the effective action of the gauge fields. This action takes the form

$$\mathcal{L}_{\text{ferm}} = k_2 \Gamma(A^{(1)}(0)) + k_1 \Gamma(A^{(2)}(0)) + \Gamma(U(1)), \tag{3.2}$$

where the $U(1)$ is the relative $U(1)$ in $U(k_1) \times U(k_2)$, and $\Gamma(A)$ is the Polyakov-Wiegmann action [6] corresponding to the relevant gauge group. The overall $U(1)$ does not appear in $\mathcal{L}_{\text{ferm}}$ since it does not couple to the fermions. $\Gamma(U(1))$ is the action (2.2) with traces taken over the two gauge groups. The $U(1) \times U(1)$ part of the dynamics decouples from the non-Abelian parts. It is identical to the discussion in the previous section. So we ignore it below, and focus on the $SU(k_1) \times SU(k_2)$ part.

We need the non-Abelian generalization of \mathcal{L}_{kin} (2.13) and $\mathcal{L}_{\text{inflow}}$ (2.11). The kinetic terms for the gauge fields are simple generalizations of (2.13). The generalization of (2.5) is

$$\int F_3 \wedge CS(A^{(i)}) , \tag{3.3}$$

where $CS(A)$ is the Chern-Simons form,

$$CS(A^{(i)}) \equiv \text{Tr} \left(A^{(i)} \wedge F^{(i)} + \frac{2}{3} A^{(i)} \wedge A^{(i)} \wedge A^{(i)} \right) . \tag{3.4}$$

There are k_1 units of F_3 flux through the sphere labeled by Ω_v (2.1), and k_2 units of F_3 through the sphere Ω_u . Integrating over the three-spheres, as in the previous section, we find that the non-Abelian part of \mathcal{L}_{CS} reads

$$\mathcal{L}_{\text{CS}} = k_2 CS(A^{(1)}) + k_1 CS(A^{(2)}) . \tag{3.5}$$

The generalization of (2.8) is

$$\int F_5 \wedge \text{Tr} A^{(i)} . \tag{3.6}$$

Therefore, F_5 only couples to the $U(1)$ parts of the gauge groups, and can be ignored in the present discussion. Note that the Chern-Simons terms do not mix the non-Abelian gauge fields on the two branes. The $SU(k_1) \times SU(k_2)$ anomalies cancel between the fermion contribution (3.2) and the Chern-Simons term (3.5).

The low energy dynamics breaks up into two decoupled problems, one for $SU(k_1)$ and the other for $SU(k_2)$. Thus, it is enough to discuss one of them, say that of $SU(k_1)$. The effective two dimensional lagrangian for the gauge field $A_\mu^{(1)}$ is

$$\mathcal{L}_1 = k_2 \Gamma(A^{(1)}(0)) + k_2 \int_0^\infty du \, CS(A^{(1)}) + \frac{1}{g_1^2} \text{Tr} \int_0^\infty du u^3 \left[\frac{1}{2} (F_{+-}^{(1)})^2 - F_{u+}^{(1)} F_{u-}^{(1)} \right] . \tag{3.7}$$

The lagrangian is no longer quadratic in the fields, but it is still instructive to look at the solutions of the equations of motion. Again we are interested in massless excitations in two dimensions, so we set (as in (2.19) and the discussion following it)

$$F_{+-}^{(1)}(u) = 0, \quad D_- F_{u+}^{(1)} = D_+ F_{u-}^{(1)} = 0, \tag{3.8}$$

where D stands for covariant derivatives in $SU(k_1)$. The analog of the bulk equations of motion, (2.15), (2.16) is

$$\begin{aligned} \frac{1}{g_1^2} D_u \left(u^3 F_{u-}^{(1)} \right) - 2k_2 F_{u-}^{(1)} &= 0, \\ \frac{1}{g_1^2} D_u \left(u^3 F_{u+}^{(1)} \right) + 2k_2 F_{u+}^{(1)} &= 0 . \end{aligned} \tag{3.9}$$

To simplify these equations it is convenient to choose the gauge $A_u^{(1)} = 0$, such that $D_u = \partial_u$ and $F_{u\pm}^{(1)} = \partial_u A_\pm^{(1)}$. As in section 2, we conclude that $F_{u+}^{(1)} = 0$ and

$$A_-^{(1)} = h_-(x^-) e^{-\frac{g_1^2 k_2}{u^2}} . \tag{3.10}$$

We see that the $SU(k_1)$ dynamics describes a chiral $k_1 \times k_1$ matrix $h_-(x^-)$, which lives at $u \sim g_1 \sqrt{k_2}$. In contrast to section 2, this matrix does not consist of free fields. The non-Abelian dynamics of (3.7) gives rise to interactions, such that h_- in fact describes a chiral $SU(k_1)$ WZW model at level k_2 . Similarly, the $SU(k_2)$ dynamics gives rise to a chiral $k_2 \times k_2$ matrix living at $v \sim g_2 \sqrt{k_1}$, describing a chiral WZW model $SU(k_2)_{k_1}$ (see figure 2b).

The fact that the $SU(k_1)$ and $SU(k_2)$ currents commute, and can be completely decoupled can be seen in the gauge theory as follows. Varying the lagrangian (3.7) with respect to the boundary variables, we get an analog of equation (2.22), but with zero on the left hand side. The reason is that the origin of the left hand side in (2.22) is \mathcal{L}_{mix} (2.10), which is absent in the non-Abelian case. Thus, the non-Abelian analog of the function $g(x^-)$, defined in (2.22), vanishes and the two non-Abelian parts of the current algebra do not couple to each other.

Note that the fact that the $SU(k_1)$ and $SU(k_2)$ currents commute and live at different locations in the plane (figure 2b) does not mean that the two chiral WZW models, $SU(k_1)_{k_2}$ and $SU(k_2)_{k_1}$ are completely decoupled. In fact, each of them separately is not a consistent quantum field theory. Many physical observables, such as the fermions Ψ_n^a , must have support in all parts of the (u, v) plane in which the currents are localized, since only when we combine the contributions from the two $SU(k_i)$ and from $U(1)$ do we get single valued observables in two dimensions.

To summarize, we arrive at the following picture. At $g_s = 0$, the $k_1 k_2$ chiral fermions Ψ_n^a are located at the intersection, $u = v = 0$, as suggested by their description in terms of open strings stretched between $D5$ -branes. Turning on the coupling constant does not lift them (which is impossible due to their chirality), but rather moves them away from the origin. Different currents constructed out of the fermions move in different directions (in the way depicted in figure 2). The $SU(k_1)_{k_2}$ currents move to $u \sim g_1 \sqrt{k_2}$. The $SU(k_2)_{k_1}$ ones move to $v \sim g_2 \sqrt{k_1}$. The $U(1)$ part is supported in both regions.

The fermions themselves, which can be thought of as solitons constructed out of the currents, become delocalized, “fat,” objects, with support in both regions mentioned in the previous paragraph. Thus, the main physical effect of the anomaly inflow is to delocalize the two dimensional physics in the higher dimensional space.

Focusing on the region near the intersection, $u \ll g_1, v \ll g_2$, we find no massless degrees of freedom. In this region, the extreme IR theory on the first type of fivebranes is $SU(k_1)$ Chern-Simons theory at level k_2 , living on the half plane $u \geq 0$, while that on branes of the second type has gauge group $SU(k_2)$ and level k_1 , and lives on the half plane $v \geq 0$. The two boundaries of the half-planes, $u = v = 0$ are identified (see figure 1).

Level-rank duality implies that the two Chern-Simons theories living on the u and v half-planes are the same, so in fact the infrared theory can be described as a *single* Chern-Simons theory (either $SU(k_1)_{k_2}$ or $SU(k_2)_{k_1}$) living on the whole plane, or, after including time, on $\mathbb{R}^{2,1}$.

3.2 Separated fivebranes

In this subsection we discuss the modification of the gauge theory picture when the fivebranes are separated in the transverse \mathbb{R}^4 's. The main effect of the separation is to give masses to some of the gauge bosons, generically breaking the gauge symmetry from $U(k_i) \rightarrow U(1)^{k_i}$. For concreteness, and to facilitate the detailed comparison to the closed string analysis of the next section, we will focus on a point in moduli space, at which each set of fivebranes is arranged symmetrically around a circle of radius r in the transverse \mathbb{R}^4 . It is not difficult to generalize the analysis to more general fivebrane configurations; the qualitative picture is independent of the precise pattern of separations.

In the $NS5$ -brane configuration we will discuss, the masses of all the W-bosons are comparable, of order $M_W \simeq r M_s^2 / g_s$. We will assume that $M_W \ll M_s$. This corresponds to very small separations of the fivebranes, $r \ll g_s l_s$. The regime $M_W \gg M_s$ will be analyzed in the next section, by studying the geometry created by the fivebranes.

How does a small M_W influence the picture arrived at in the previous subsection? We expect that different parts of the current algebra will react differently to the deformation. Since a $U(1)^{k_1-1}$ out of the $SU(k_1)$ that lives at $u \sim g_1 \sqrt{k_2}$ is unbroken, we expect it to remain where it was, while the rest of the currents, parameterizing the coset $SU(k_1)_{k_2}/U(1)^{k_1-1}$ will presumably react in some way. The same should be true for $SU(k_2)_{k_1}$.

To analyze this problem precisely, one must solve the full non-Abelian gauge theory for finite M_W . This is difficult, among other things because after separating the fivebranes one can no longer reduce the problem to three dimensions, as we have done in the previous subsection. Rather, one has to study the full six dimensional theory on each set of fivebranes.

Here we will content ourselves by studying the theory at large distances, at which the massive gauge bosons are irrelevant, and one can restrict to the dynamics of the Abelian gauge theory, $U(1)^{k_1+k_2}$. This description is expected to be valid for $u, v \gg 1/M_W$. Thus, it can provide information only about the long distance behavior of the various modes.

At distances much larger than $1/M_W$, the analysis of the previous subsection goes through, the only difference being that we take the gauge fields $A^{(i)}$ to be diagonal matrices. For example, $h_-(x^-)$ in (3.10) is a diagonal $k_1 \times k_1$ matrix, with eigenvalues $h_n(x^-)$, $n = 1, 2, \dots, k_1$. Similarly, the corresponding $k_2 \times k_2$ matrix for $A^{(2)}$ is diagonal with eigenvalues $\tilde{h}_a(x^-)$, $a = 1, \dots, k_2$.

Thus, in this case we have $k_1 k_2$ copies of the analysis of section 2. In particular, for every pair (n, a) we find a single chiral degree of freedom, which can be interpreted as $h_n(x^-)$ or as $\tilde{h}_a(x^-)$. The two are identified for the same reason as in section 2. From the point of view of the original fermions Ψ_n^a , these degrees of freedom are the $U(1)$ currents

$$J_n^a = (\Psi^*)_a^n \Psi_n^a . \tag{3.11}$$

They correspond to wavefunctions that decay like $1/u^3$ and $1/v^3$ at large distances from the intersection, as in section 2.

As mentioned in the beginning of this subsection, we expect the currents (3.11) to split naturally into a number of groups,

$$U(1) \times U(1)^{k_1-1} \times U(1)^{k_2-1} \times \left[\frac{SU(k_1)_{k_2}}{U(1)^{k_1-1}} \times \frac{SU(k_2)_{k_1}}{U(1)^{k_2-1}} \right] . \tag{3.12}$$

The first three factors in (3.12) correspond to the unbroken parts of the gauge symmetry, while the factor in square brackets is the part of the theory that is expected to react to the separation of the branes.

In terms of the currents J_n^a , (3.11), the different factors in (3.12) can be written as follows. The $U(1)^{k_1+k_2-1}$ currents can be written as

$$U(1) : \quad \sum_{n,a} J_n^a ,$$

$$U(1)^{k_1-1} : \quad \sum_{n,a} J_n^a e^{\frac{2\pi i n(2j_1+1)}{k_1}} ,$$

$$U(1)^{k_2-1} : \quad \sum_{n,a} J_n^a e^{\frac{2\pi i a(2j_2+1)}{k_2}}, \quad (3.13)$$

where

$$2j_i = 0, 1, 2, \dots, k_i - 2. \quad (3.14)$$

The remaining currents, corresponding to the cosets in the square brackets in (3.12), are the combinations of the J_n^a that are orthogonal to all those in (3.13). A convenient basis for them is

$$\tilde{J}_{j_1, j_2} = \sum_{n,a} J_n^a e^{\frac{2\pi i n(2j_1+1)}{k_1}} e^{\frac{2\pi i a(2j_2+1)}{k_2}}, \quad (3.15)$$

where j_i run over the range (3.14).

The splitting of the $k_1 k_2$ currents J_n^a into the groups (3.12) is natural from the point of view of our gauge theory analysis. Different groups correspond to different asymptotic decay of the field strengths $F^{(1)}$ at large u and $F^{(2)}$ at large v . The overall $U(1)$ degree of freedom corresponds to

$$F^{(1)}(u) \sim \frac{1}{u^3}; \quad F^{(2)}(v) \sim \frac{1}{v^3}, \quad (3.16)$$

the $U(1)^{k_1-1}$ currents (3.13) correspond to

$$F^{(1)}(u) \sim \frac{1}{u^3}; \quad F^{(2)}(v) \sim \frac{1}{v^{2j_1+4}}, \quad (3.17)$$

the $U(1)^{k_2-1}$ currents correspond to

$$F^{(1)}(u) \sim \frac{1}{u^{2j_2+4}}; \quad F^{(2)}(v) \sim \frac{1}{v^3}, \quad (3.18)$$

and the remaining currents (3.15) have a faster than inverse cubed decay in both u and v

$$F^{(1)}(u) \sim \frac{1}{u^{2j_2+4}}; \quad F^{(2)}(v) \sim \frac{1}{v^{2j_1+4}}. \quad (3.19)$$

It should be noted that the precise form of the asymptotic fall-off (3.16) – (3.19) is valid for the particular point in the moduli space where the fivebranes are placed on circles in the transverse \mathbb{R}^4 's, but the basic fact that the $U(1)^{k_1+k_2-1}$ currents correspond to field strengths that decay like $1/u^3$ and/or $1/v^3$, while the factor in square brackets in (3.12) corresponds to field strengths with more rapid decay in both u and v , is general.

It is also worth reiterating that the Abelian analysis above only captures the behavior of the different modes for large u , v . It is not sensitive enough to see how figure 2 gets deformed as we turn on a small M_W ; that requires tools with much higher resolution. In particular, the fact that the factor in square brackets in (3.12) consists (for $NS5$ -branes) of a part that lives near $v = l_s \sqrt{k_1}$, and a part that lives near $u = l_s \sqrt{k_2}$, as is expected from the analysis that led to figure 2, cannot be resolved at this level. The Abelian analysis combines these two parts together and replaces them by the $(k_1-1)(k_2-1)$ currents (3.15).

4. Closed string description

The supergravity background corresponding to the configuration of intersecting $NS5$ -branes discussed in the previous sections factorizes into three decoupled parts [7]. The $\mathbb{R}^{1,1}$, labeled by x^\pm , which is common to both sets of branes, is trivial. The four dimensional space labeled by (x^2, x^3, x^4, x^5) is described by the CHS solution [8] associated with the k_2 fivebranes stretched in (016789). Similarly, the supergravity background in the four dimensional space labeled by (x^6, x^7, x^8, x^9) is that associated with the fivebranes stretched in (012345). The fact that the background factorizes will play an important role in our considerations below.

To present the solution, we define

$$\begin{aligned} \mathbf{y} &= (x^2, x^3, x^4, x^5) , \\ \mathbf{z} &= (x^6, x^7, x^8, x^9) . \end{aligned} \tag{4.1}$$

We have k_1 fivebranes localized at the points $\mathbf{z} = \mathbf{z}_n$, $n = 1, 2, 3, \dots, k_1$, and k_2 fivebranes localized at $\mathbf{y} = \mathbf{y}_a$, $a = 1, 2, 3, \dots, k_2$. Every pair of fivebranes from different sets intersects at a point in \mathbb{R}^8 , $(\mathbf{y}, \mathbf{z}) = (\mathbf{y}_a, \mathbf{z}_n)$.

The geometry corresponding to this brane configuration has the product form

$$\mathbb{R}^{1,1} \times \text{CHS}_{\mathbf{y}} \times \text{CHS}_{\mathbf{z}} . \tag{4.2}$$

The supergravity background is

$$\begin{aligned} \Phi &= \Phi_1(\mathbf{z}) + \Phi_2(\mathbf{y}), \\ g_{\mu\nu} &= \eta_{\mu\nu}, \quad \mu, \nu = 0, 1 , \\ g_{\alpha\beta} &= e^{2(\Phi_2 - \Phi_2(\infty))} \delta_{\alpha\beta}, \quad H_{\alpha\beta\gamma} = -\epsilon_{\alpha\beta\gamma\delta} \partial^\delta \Phi_2, \quad \alpha, \beta, \gamma, \delta = 2, 3, 4, 5, \\ g_{pq} &= e^{2(\Phi_1 - \Phi_1(\infty))} \delta_{pq}, \quad H_{pqr} = -\epsilon_{pqrs} \partial^s \Phi_1, \quad p, q, r, s = 6, 7, 8, 9, \end{aligned} \tag{4.3}$$

where

$$\begin{aligned} e^{2(\Phi_1 - \Phi_1(\infty))} &= 1 + \sum_{n=1}^{k_1} \frac{l_s^2}{|\mathbf{z} - \mathbf{z}_n|^2}, \\ e^{2(\Phi_2 - \Phi_2(\infty))} &= 1 + \sum_{a=1}^{k_2} \frac{l_s^2}{|\mathbf{y} - \mathbf{y}_a|^2} . \end{aligned} \tag{4.4}$$

In the rest of this section we describe some properties of this background, first for the case of coincident fivebranes, $\mathbf{z}_n = \mathbf{y}_a = 0$, and then for separated ones.

4.1 Coincident fivebranes

This is a particularly symmetric case, in which the brane configuration preserves $SO(4)_{2345} \times SO(4)_{6789}$. Defining $u = |\mathbf{y}|$, $v = |\mathbf{z}|$ and setting $\alpha' = 2$, the geometry (4.3) takes the form

$$\begin{aligned} ds^2 &= -(dx^0)^2 + (dx^1)^2 + f_1(v)(dv^2 + v^2 d\Omega_v^2) + f_2(u)(du^2 + u^2 d\Omega_u^2), \\ e^{2(\Phi - \Phi(\infty))} &= f_1(v)f_2(u), \quad f_1(v) = 1 + \frac{2k_1}{v^2}, \quad f_2(u) = 1 + \frac{2k_2}{u^2}. \end{aligned} \tag{4.5}$$

There is also a flux of the NS B field through the two three-spheres. As $u, v \rightarrow \infty$, the fivebrane background (4.5) approaches flat spacetime (2.1).

As mentioned above, the intersecting fivebrane geometry contains two copies of the CHS solution corresponding to a single stack of fivebranes. It has two throats, one associated with u , the other with v . The string coupling becomes large when we go down either of these throats. Note also that since each of the CHS backgrounds associated with \mathbf{y} and \mathbf{z} corresponds to an exact solution of the classical string equations of motion (*i.e.* it is a worldsheet CFT with central charge $c = 6$), the factorization (4.2) is valid in the full classical string theory and is not restricted to the supergravity approximation.

To focus on the physics near the intersection of the two sets of fivebranes we take the near-horizon limit. This is done in the standard way [9],[4] by rescaling the variables v and u by $\exp(\Phi_1(\infty))$ and $\exp(\Phi_2(\infty))$, respectively, and sending $\exp(\Phi_i(\infty)) \rightarrow 0$. This means that we are focusing on the region $u, v \sim g_s l_s$ in the full geometry, in the limit $g_s \rightarrow 0$. The resulting near-horizon geometry is

$$\mathbb{R}^{1,1} \times \mathbb{R}_{\phi_1} \times SU(2)_{k_1} \times \mathbb{R}_{\phi_2} \times SU(2)_{k_2} . \tag{4.6}$$

where $\phi_1 = \sqrt{2k_1} \ln v$ and $\phi_2 = \sqrt{2k_2} \ln u$. \mathbb{R}_{ϕ_i} has a linear dilaton with

$$Q_i = \sqrt{\frac{2}{k_i}} . \tag{4.7}$$

Hence the worldsheet central charge of ϕ_i is $c_i = 1 + 3Q_i^2$. The $SU(2)$ WZW models correspond to the angular three-spheres labeled by Ω_v and Ω_u , respectively. The $SO(4) \times SO(4)$ symmetry of the brane configuration is realized as the symmetry group of the two $SU(2)$ CFT's in (4.6).

Note that the above discussion is only valid when both k_1 and k_2 are larger than one. When $k_i = 1$ the throat labeled by ϕ_i is absent and we expect the description of the intersection to be modified. We will return to this point in the discussion.

As mentioned in the introduction, the near-horizon geometry (4.6) exhibits an unexpected enhanced symmetry. Performing the rotation (see figure 3)

$$\begin{aligned} Q\phi &= Q_1\phi_1 + Q_2\phi_2 , \\ Qx^2 &= Q_2\phi_1 - Q_1\phi_2 , \end{aligned} \tag{4.8}$$

where

$$Q = \sqrt{\frac{2}{k}}; \quad \frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} , \tag{4.9}$$

we find that (4.6) takes the form (1.3),

$$\mathbb{R}^{2,1} \times \mathbb{R}_{\phi} \times SU(2)_{k_1} \times SU(2)_{k_2} ,$$

which is indeed invariant under $ISO(2,1)$. This symmetry enhancement is surprising since the physical origin of the $\mathbb{R}^{1,1}$ factor, which consists of directions along the fivebranes, and of the x^2 direction, which is a combination of directions transverse to one of the two

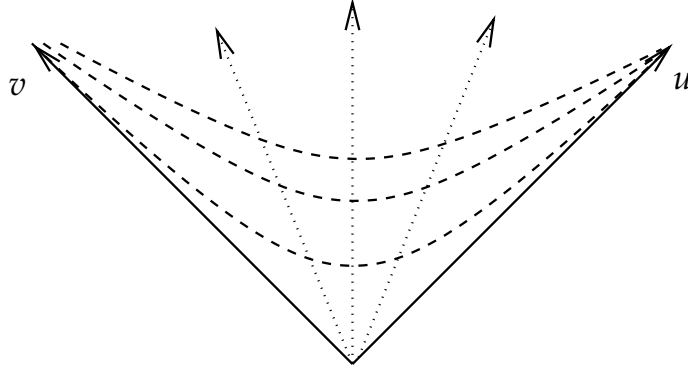


Figure 3: The dashed lines represents constant ϕ contours that are parameterized by x^2 . The dotted lines are constant x^2 contours that are parameterized by ϕ .

branes, is completely different. Apparently, in the vicinity of the intersection, the two become indistinguishable.

The enhanced Poincare symmetry implies also a higher supersymmetry. Let us briefly recall how this comes about. Thinking of the near-horizon geometry as a product of an $\mathbb{R}^{1,1}$ and two CHS geometries, as in (4.6), one can construct in the usual way eight spacetime supercharges, by using the worldsheet superpartners of the coordinates on the spacetime (4.6). These are ten free fermions: two fermions ψ_{\pm} corresponding to $\mathbb{R}^{1,1}$, two more that correspond to the two Liouville modes, ψ_1, ψ_2 , three fermions in the adjoint of $SU(2)_{k_1}$, $\lambda_a^{(1)}$, $a = 3, \pm$, and three in the adjoint of $SU(2)_{k_2}$, $\lambda_a^{(2)}$. To construct the spacetime supercharges, it is convenient to bosonize the fermions as follows:

$$\begin{aligned}
 \psi_{\pm} &\rightarrow H_1, \\
 \psi_1, \lambda_3^{(1)} &\rightarrow H_2, \\
 \lambda_{\pm}^{(1)} &\rightarrow H_3, \\
 \psi_2, \lambda_3^{(2)} &\rightarrow H_4, \\
 \lambda_{\pm}^{(2)} &\rightarrow H_5,
 \end{aligned}
 \tag{4.10}$$

and similarly for the other worldsheet chirality.

The following spacetime supercharges can be checked to be unbroken in the background (4.6):

$$\begin{aligned}
 Q_+ &= \oint \frac{dz}{2\pi i} e^{-\frac{\varphi}{2} + \frac{i}{2} H_1 \pm \frac{i}{2} (H_2 + H_3) \pm \frac{i}{2} (H_4 + H_5)}, \\
 \bar{Q}_+ &= \oint \frac{d\bar{z}}{2\pi i} e^{-\frac{\bar{\varphi}}{2} + \frac{i}{2} \bar{H}_1 \pm \frac{i}{2} (\bar{H}_2 - \bar{H}_3) \pm \frac{i}{2} (\bar{H}_4 - \bar{H}_5)}.
 \end{aligned}
 \tag{4.11}$$

The first line comes from the left-moving sector on the worldsheet, while the second comes from the right-movers. Note that (4.11) contains eight supercharges, all with the same chirality in $\mathbb{R}^{1,1}$. Note also that the left and right-moving supercharges have opposite chiralities under $SO(4)_{2345}$ and $SO(4)_{6789}$. This is a well known property of $NS5$ -branes [8].

The spacetime supercharges (4.11) have the nice property that they remain good symmetries when we deform the system by separating the fivebranes, and when we go beyond the near-horizon limit and study the full fivebrane geometry (4.5). However, in the near-horizon geometry (4.6), we expect to find twice as many supercharges, due to the enhanced Lorentz symmetry of the background (1.3).

A quick way to construct the extra supercharges is to apply to the ones we already have, (4.11), the generators of Lorentz transformations that mix x^2 (4.8) with x^\pm . The worldsheet superpartner of x^2 is ψ_{x^2} ,

$$Q\psi_{x^2} = Q_2\psi_1 - Q_1\psi_2 . \tag{4.12}$$

The Lorentz generators that mix it with ψ_\pm have the form

$$J_{2\pm} = \oint \psi_{x^2}\psi_\pm + \oint \bar{\psi}_{x^2}\bar{\psi}_\pm . \tag{4.13}$$

Applying them to the supercharges (4.11) one finds eight more supercharges, with the opposite chirality in $\mathbb{R}^{1,1}$. Since (4.13) is a symmetry of the background (1.3), the resulting supercharges must be good symmetries as well, a fact that can be verified directly. Altogether, we find sixteen supercharges which transform as $(2, 2, 2) \oplus (2, \bar{2}, \bar{2})$ under $ISO(2, 1) \times SO(4) \times SO(4)$. It is easy to work out the supersymmetry algebra of these supercharges. One unusual aspect of this superalgebra is that the anticommutator of two supercharges includes the $SO(4) \times SO(4)$ R-symmetry generators. This is to be distinguished from the standard extended supersymmetry algebras which do not include such generators. This point was independently noticed from another point of view in [1].

The near-horizon geometry of intersecting fivebranes (1.3), (4.6) can be viewed as a 2+1 dimensional vacuum of Little String Theory (LST) [10],[11] with sixteen supercharges. Thus, in studying it one can use the techniques developed in recent years for studying such vacua; see *e.g.* [5],[12][13][14]-[15]. In particular, one can use the geometry to classify off-shell observables in the theory, which correspond to non-normalizable operators, whose wavefunctions are peaked at $\phi \rightarrow \infty$. One can also use it to study bulk physics of delta-function normalizable modes that live in the throat.

The fact that the string coupling diverges as $\phi \rightarrow -\infty$ means that we cannot use the geometry to study states that are localized there, or calculate generic correlation functions of the non-normalizable observables, which receive contributions from the strong coupling region. However, experience from other vacua of LST suggests that the physics associated with the strong coupling region can often be at least partially understood by studying the low energy field theory on the branes that give rise to the linear dilaton background.

In our case, that theory was analyzed in section 3, and it is natural to ask how the picture found there compares to the one suggested by the geometry. The analysis in section 3.1 led to the picture summarized in figure 2. The low energy spectrum consists of three components, (1.2), each of which is located at a different place. We would like to understand the interpretation of these modes in the geometry (4.5), and its near-horizon limit (1.3).

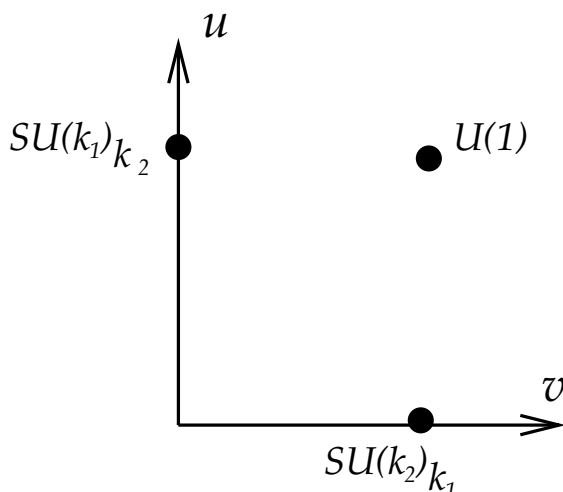


Figure 4: The location of the various currents in the strong coupling description.

Consider first the $U(1)$ mode. In the gauge theory discussion of section 2 it had support at $u \sim l_s \sqrt{k_2}$ and $v \sim l_s \sqrt{k_1}$. From the higher dimensional perspective these two points are $(u, v) \sim (l_s \sqrt{k_2}, 0)$ and $(u, v) \sim (0, l_s \sqrt{k_1})$. Below we show that in the strong coupling limit the support is at $(u, v) \sim (l_s \sqrt{k_2}, l_s \sqrt{k_1})$ (see figure 4).

In the geometric description, this mode corresponds to an excitation of the RR field strength $F = F_3 + F_5 + F_7$, of the form

$$F \sim dP \wedge (H_3^y + \star_y H_3^y) \wedge (H_3^z + \star_z H_3^z) . \quad (4.14)$$

Here P is a chiral scalar field living in $\mathbb{R}^{1,1}$, H_3^y and H_3^z are the field strengths of the NS B -field given in (4.3), and \star_y and \star_z are the Hodge duals in the four dimensional spaces labeled by y and z , respectively.

Eq. (4.14) describes a normalizable mode whose wavefunction is localized in the transition region between the throats in u , v , and the asymptotically flat space far from the fivebranes. At large u , v the five-form part of (4.14) falls off like

$$F_5 = dP \wedge \left(\frac{1}{v^3} dv \wedge d\Omega_u + \frac{1}{u^3} du \wedge d\Omega_v \right) . \quad (4.15)$$

Note that both terms in (4.15) fall off like $u^{-3}v^{-3}$ at large u , v . For example, in the first term the fall-off in v is manifest, while the one in u is due to the fact that the metric on the three-sphere labeled by Ω_u (4.5) goes like $ds^2 = u^2 d\Omega_u^2$ for large u .

The behavior (4.15) is in precise agreement with the gauge theory analysis of sections 2, 3. Indeed, there it was found that the chiral boson describing the $U(1)$ degree of freedom in (1.2) corresponds to gauge field strength $F_{u-}^{(1)}$ on brane 1 that falls off like $1/u^3$ (2.20), and field strength on brane 2, $F_{v-}^{(2)}$, that falls off like $1/v^3$. To compare to (4.15) one needs to recall that the gauge fields on the fivebranes correspond to RR operators in the geometry (see *e.g.* [15]). Thus, the first term in parenthesis in (4.15) corresponds to $\text{Tr} F_{v-}^{(2)}$, while

the second corresponds to $\text{Tr}F_{u-}^{(1)}$. The fact that the full solution (4.14) includes both of these terms is the closed string analog of the fact that in our gauge theory analysis we found that the $U(1)$ mode involves both $A^{(1)}$ and $A^{(2)}$.

The left-moving scalar field P (4.14) lives outside of the near-horizon throat. It is a singleton, that does not suffer from the strong coupling problem of the background (4.5). This agrees with the gauge theory picture, where this $U(1)$ degree of freedom is free and decoupled from the non-Abelian dynamics governing the rest of (1.2).

What about the non-Abelian parts of the current algebra (1.2)? Consider for example the $SU(k_1)_{k_2}$ currents, which according to figure 4 live at $(u \simeq l_s \sqrt{k_2}, v = 0)$. From the point of view of the geometry (4.5) they correspond to modes of the RR field strengths which live in the transition region between the throat and asymptotically flat space in \mathbf{y} , and in the strong coupling region of the throat in \mathbf{z} . Thus, they cannot be studied by using supergravity or classical string theory. From the closed string perspective, the strong coupling effects are due to the fact that the non-Abelian dynamics that gives rise to the interacting $SU(k_1)_{k_2}$ WZW model involves D-branes that descend down the throat labeled by v , and become light there. This should be contrasted with the $U(1)$ current, which in the gauge theory is free, and in the closed string description corresponds to a mode that lives outside both throats.

The comparison of the field theory analysis of sections 2, 3 to the analysis of the geometry of the fivebranes provides us with a more detailed understanding of the emergence of the enhanced super-Poincare symmetry of the near-horizon geometry (1.3). We see that in the full problem we have $1 + 1$ dimensional chiral modes, which break $ISO(2, 1)$, but as we take the near-horizon limit, they go off to infinity in different directions. The $U(1)$ part goes to the weakly coupled boundary $\phi \rightarrow \infty$, becomes a singleton, and decouples. The $SU(k_1)_{k_2}$ and $SU(k_2)_{k_1}$ parts go to $x^2 \rightarrow -\infty$ and $x^2 \rightarrow \infty$, respectively. We are left with no light $1 + 1$ dimensional degrees of freedom. The extreme IR behavior of the theory is described by the Chern-Simons theory discussed in section 3, which from the closed string point of view lives in the strong coupling region of (1.3).

4.2 Separated fivebranes

A well known way to avoid the strong coupling singularity of the coincident fivebrane background (4.5), is to study the theory along its Coulomb branch. The near-horizon description is particularly simple for the case where the two stacks of fivebranes are placed symmetrically on circles in the respective transverse \mathbb{R}^4 's. Parameterizing $(\mathbb{R}^4)_{6789}$ by the complex coordinates (a_1, b_1) and $(\mathbb{R}^4)_{2345}$ by (a_2, b_2) , we place the first set of fivebranes at

$$(a_1^{(n_1)}, b_1^{(n_1)}) = (0, r_1 e^{\frac{2\pi i n_1}{k_1}}); \quad n_1 = 1, 2, \dots, k_1, \quad (4.16)$$

and the second set at

$$(a_2^{(n_2)}, b_2^{(n_2)}) = (0, r_2 e^{\frac{2\pi i n_2}{k_2}}); \quad n_2 = 1, 2, \dots, k_2. \quad (4.17)$$

Here r_j are the radii of the two circles on which the fivebranes are located. In the gauge theory on the fivebranes, (a_j, b_j) correspond to scalar fields in the adjoint of $U(k_j)$, (A_j, B_j) . The configuration (4.16), (4.17) corresponds to a point in the Coulomb branch of the gauge theory at which

$$\langle B_j \rangle = r_j \text{diag}(e^{\frac{2\pi i}{k_j}}, e^{\frac{4\pi i}{k_j}}, \dots, e^{\frac{2\pi i(k_j-1)}{k_j}}, 1) . \quad (4.18)$$

The masses of the broken gauge bosons are set by the radii of the circles, $M_W^{(j)} = r_j M_s^2 / g_s$.

The background (4.18) is a simple generalization of those studied for a single stack of fivebranes in [5],[13][14]-[15], and we can use the results of these papers in analyzing it. When $M_W^{(j)} \gg M_s$, the string background corresponding to the configuration (4.18) is perturbative. The separation of the fivebranes eliminates the strong coupling singularity at the origin, and the string coupling remains small everywhere. The near-horizon geometry (4.6) is replaced after the deformation by

$$\mathbb{R}^{1,1} \times \left(\frac{SL(2)_{k_1}}{U(1)} \times \frac{SU(2)_{k_1}}{U(1)} \right) / Z_{k_1} \times \left(\frac{SL(2)_{k_2}}{U(1)} \times \frac{SU(2)_{k_2}}{U(1)} \right) / Z_{k_2} . \quad (4.19)$$

It involves two cigar CFT's, describing the two regulated throats of (4.6). The string coupling in the background (4.19) is bounded from above by $g_s^2 = M_s^2 / (M_W^{(1)} M_W^{(2)})$, and therefore is small everywhere. It attains its maximum value at the tip of both cigars.

Since in this case the fivebrane background is weakly coupled everywhere, we should be able to see all $k_1 k_2$ chiral currents using perturbative string theory. We next show that this is indeed the case.

We would like to compute the spectrum of massless excitations in 1 + 1 dimensions in the background (4.19). Using the techniques of [5],[13][14]-[15] it can be checked that the only sector that can contain such states is the RR sector. All the other sectors have a finite mass gap. Since the background breaks up into a product of three factors, the problem of finding massless modes breaks up into the problem of finding normalizable zero modes in the different factors. A detailed discussion of this problem appears in appendix A. Here we only summarize the results.

Due to the product structure of (4.19), it is enough to focus on one of the

$$\left(\frac{SL(2)_k}{U(1)} \times \frac{SU(2)_k}{U(1)} \right) / Z_k \quad (4.20)$$

factors. The spectrum of massless RR states on (4.20) is known since it plays an important role in analyzing the six dimensional system of k IIB fivebranes on a circle. In that case, the massless RR operators correspond holographically to the operators

$$\text{Tr} F_{\mu\nu} B^{2j+1} \quad (4.21)$$

in the low energy $SU(k)$ gauge theory [15]. When B has an expectation value of a form analogous to (4.18), the operators (4.21) create states associated with combinations of the unbroken $U(1)^{k-1}$ gauge bosons

$$|O_j\rangle = \sum_{n=1}^k e^{\frac{2\pi i n(2j+1)}{k}} F_{\mu\nu}^{(n)} |0\rangle . \quad (4.22)$$

In the near-horizon geometry (4.20), these states correspond to normalizable wavefunctions, O_j , that behave near the boundary $\phi \rightarrow \infty$ like $\exp(-Qj\phi)$, with j running over the same range as in (3.14). If we include the asymptotically flat part of the space and denote the asymptotic radial direction by u , one can show that for large u (far outside the fivebrane throat) the wavefunctions O_j behave like

$$O_j \simeq \frac{1}{u^{2j+4}} . \quad (4.23)$$

Of course, in addition to the $k - 1$ wavefunctions O_j which correspond to the Cartan subalgebra of $SU(k)$, the full fivebrane geometry includes the wavefunction $O_{-\frac{1}{2}}$, which corresponds to the overall $U(1)$ gauge boson (4.22). As mentioned earlier, this wavefunction lives in the transition region between the throat and the flat space far from the branes, and does not correspond to a normalizable mode in the near-horizon geometry. Altogether we have k normalizable wavefunctions living in the full fivebrane geometry, of which $k - 1$ live in the throat (4.20), and one in the transition region.

Given the above spectrum of massless RR modes in the throat of a single stack of fivebranes, it is not difficult to find the corresponding spectrum in the geometry of the intersecting ones (4.19). We simply take a product of any of the k_1 wavefunctions which live in the throat of the first stack of fivebranes, with any of the k_2 wavefunctions which live in the throat of the second stack. We find $k_1 k_2$ chiral massless modes (see appendix A). These modes naturally split into the following groups:

- (1) One mode supported in the transition region of both throats in (4.19).
- (2) $k_1 - 1$ modes supported in the transition region of throat 2, and deep inside throat 1.
- (3) $k_2 - 1$ modes supported in the transition region of throat 1, and deep inside throat 2.
- (4) $(k_1 - 1)(k_2 - 1)$ modes supported deep in both throats 1, 2.

Only the last class corresponds to normalizable modes in the near-horizon geometry (4.19). In analogy with (4.21), they can be thought of as states created by the operators

$$\text{Tr} \Psi B_1^{2j_1+1} \Psi^* B_2^{2j_2+1} , \quad (4.24)$$

acting on the vacuum.

Note that the set of modes found in the geometry is in nice correspondence with those seen in the gauge theory analysis of section 3. In particular, the first three types of states described geometrically above are precisely the states (3.13), while the fourth one corresponds to (3.15), a fact that can be seen directly by substituting (4.18) into (4.24). As a check, the fall-off of the vertex operators of these states agrees with the gauge theory results (3.16) – (3.19).

5. The big picture

In sections 2–4 we analyzed some aspects of the dynamics of the system of intersecting fivebranes from the gauge theory and gravity points of view. In this section we would like to put the two pictures together.

We start with the coincident fivebrane configuration. The low energy modes are spread out in the (u, v) plane in the way depicted in figure 4. Now, imagine separating the first stack of k_1 fivebranes as in (4.16), while keeping those in the second stack coincident. In the gauge theory, this means that $M_W^{(1)} \neq 0$, while $M_W^{(2)} = 0$. We would like to describe what happens to the different modes in figure 2 as we increase $M_W^{(1)}$.

There are a number of different regimes to consider:

- (1) $M_W^{(1)} \ll \frac{1}{l_s}$.
- (2) $M_W^{(1)} \gg \frac{1}{l_s}$, $M_W^{(1)} g_s l_s^2 = r_1 \ll l_s$.
- (3) $M_W^{(1)} \gg \frac{1}{l_s}$, $M_W^{(1)} g_s l_s^2 = r_1 \gg l_s$.

Note that while regimes (1) and (2) can be studied in the near-horizon geometry, regime (3) corresponds to distances outside the near-horizon limit.

In regime (1), the picture is very similar to that seen for $M_W^{(1)} = 0$ in section 3. Since we did not separate the k_2 fivebranes of the second type, the $SU(k_2)_{k_1}$ factor in figure 4 remains intact. On the other hand, the $SU(k_1)_{k_2}$ factor that lives on the u axis in figure 4 splits into the unbroken $U(1)^{k_1-1}$, which remains where it was, and the coset $SU(k_1)_{k_2}/U(1)^{k_1-1}$, which starts moving down in figure 4, in the direction of decreasing u .

Regime (2) can be partially analyzed using the closed string picture. The CHS throat labeled by v is now cut-off, and is replaced by a cigar, in which the string coupling remains small throughout. This allows one to use the closed string description to construct the modes corresponding to $U(1)^{k_1-1}$ currents that live near the u axis. These are modes of RR fields of a form analogous to (4.14),

$$F \sim dP \wedge (H_3^y + \star_y H_3^y) \wedge \Lambda_i; \quad i = 1, 2, \dots, k_1 - 1, \quad (5.1)$$

where Λ_i are the normalizable modes in the cigar that entered the discussion of section 4.2. There, these modes were discussed for the case that $M_W^{(2)}$ is large, but clearly they exist for $M_W^{(2)} = 0$ as well, since they are localized outside the strong coupling throat region of the fivebranes of the second type. The modes Λ_i are localized at $v \sim l_s g_s$, so the $U(1)^{k_1-1}$ currents (5.1) live very close to the u axis, as in regime (1). They are not significantly influenced by the separation of the k_1 fivebranes of the first type.

The remaining part of the $SU(k_1)_{k_2}$ is the coset $SU(k_1)_{k_2}/U(1)^{k_1-1}$. For $M_W^{(1)} \ll M_s$ it is located deep in the throat labeled by v , but outside the u throat. For $M_W^{(1)} \gg M_s$ this can no longer be the case, since the coupling in the v throat is small everywhere. Thus, if the coset degrees of freedom live outside the u throat, they should be visible to a perturbative analysis. This is a problem for two separate reasons. First, we have just discussed the spectrum of massless states living near the tip of the v cigar and outside the

u throat, and that spectrum does not include the coset degrees of freedom. Second, the coset is an interacting CFT, and we do not expect to be able to see it in a weakly coupled string description.

Thus, it must be that as $M_W^{(1)}$ increases, the coset $SU(k_1)_{k_2}/U(1)^{k_1-1}$ moves down the u throat, such that by the time we reach regime 2, it is located deep inside the strongly coupled region $u \ll l_s$.

In regime 3 the distances between any two fivebranes of the first type are large, and so we have k_1 decoupled systems, each of which has one fivebrane of the first type intersecting k_2 fivebranes of the second type. In terms of figure 2, roughly speaking¹⁰ we have a set of $SU(k_2)_1$ CFT's that are widely separated in the v direction, by distances of order the separations between the fivebranes of the first type.

As one increases $M_W^{(1)}$ from regime (2) to regime (3), the following must happen. The coset $SU(k_1)_{k_2}/U(1)^{k_1-1}$, which in regime 2 is located at small u, v , moves towards larger v and combine with the $SU(k_2)_{k_1}$ to form the CFT $(SU(k_2)_1)^{k_1}$. One can check that the central charge is precisely right for this, $c_l = k_1(k_2 - 1)$. In addition, in regime (3) we have $U(1)^{k_1}$ currents living at $u \sim l_s$, as before.

So far we described the situation in the case where $M_W^{(1)} \neq 0$, but $M_W^{(2)} = 0$. It is not difficult to generalize the discussion to the case where both are non-zero. What happened before to $SU(k_1)_{k_2}$ will now happen to both factors. The $U(1)^{k_1-1}$ factor stays on the u axis, at $u \sim \sqrt{k_2}l_s$; the $U(1)^{k_2-1}$ factor stays on the v axis at $v \sim \sqrt{k_1}l_s$. The two cosets, $SU(k_1)_{k_2}/U(1)^{k_1-1}$ and $SU(k_2)_{k_1}/U(1)^{k_2-1}$ move (as we increase $M_W^{(i)}$), the first towards small u , the second towards small v . When we reach the regime $M_W^{(i)} \gg M_s$, which as we saw in section 4 has a weakly coupled closed string description, the two cosets meet and combine to form a free theory, corresponding to the $(k_1 - 1)(k_2 - 1)$ currents (3.15). Note that the fact that the two interacting coset CFT's give rise when they combine to a free theory is necessary for consistency of the spacetime dynamics and the closed string description of it, since the latter can be sent to arbitrarily weak coupling by tuning $M_W^{(i)}$.

6. Discussion

There are a number of possible extensions of the results presented in this paper. In this section we mention some of them.

6.1 Thermodynamics

One natural set of questions concerns the thermodynamics of the system of intersecting fivebranes. When all the fivebranes in a given set are coincident we found that the near-horizon geometry has the form (1.3). As usual for gravitational systems, we expect the high energy thermodynamics to be dominated by black branes. For asymptotically linear dilaton spacetime such as (1.3), the relevant black brane is

$$\frac{SL(2, \mathbb{R})_k}{U(1)} \times \mathbb{R}^2 \times SU(2)_{k_1} \times SU(2)_{k_2} . \tag{6.1}$$

¹⁰To obtain a more accurate representation of this regime, we must take into account the angular three-spheres as well.

Here the level k is related to the numbers of fivebranes k_1, k_2 via (4.9). Note that it satisfies the bound $k \geq 1$ for $k_1, k_2 \geq 2$, so it is a normalizable state in this regime (see [16],[17] for recent discussions of this bound, and the high energy thermodynamics associated with (6.1)). For $k_1 = k_2 = 2$ one has $k = 1$, which is the borderline case for normalizability.

The background (6.1) describes a non-extremal solution, in which a finite energy density is added to the fivebrane intersection. When the energy density in string units is very large, the (Euclidean) solution is weakly coupled everywhere, and can be studied using perturbative string techniques. It is interesting that this solution preserves the Euclidean group $ISO(2)$; in particular, it is translationally invariant in x^2 , and is invariant under rotations that mix x^1 and x^2 . This provides further evidence for the fact that the near-horizon dynamics of coincident fivebranes preserves $ISO(2,1)$. We have seen earlier that the low energy dynamics is consistent with this assertion, and now we see that the high energy thermodynamics is consistent with it as well.

The Bekenstein-Hawking entropy associated with the black hole (6.1) has the usual Hagedorn form

$$S_{\text{bh}} = 2\pi l_s \sqrt{k} E . \tag{6.2}$$

This can be compared with the Hagedorn growth of each set of fivebranes, which has a form similar to (6.2), with $k \rightarrow k_i$. The relation (4.9) implies that $k < k_1, k_2$, so that the Hagedorn growth associated with the intersection is smaller than that associated with each group of fivebranes separately. This had to be the case, since otherwise the system of non-extremal parallel fivebranes would develop such intersections dynamically, to increase its entropy.

6.2 $k_1 = 1$

As mentioned in section 4, our discussion of the near-horizon geometry of the intersecting fivebrane system is valid when each stack consists of two or more branes. Indeed, the geometry (1.3), (4.6) does not make sense when one of the k_i is equal to one, since it contains a negative level bosonic $SU(2)$ WZW model.

A very similar problem occurs for the case of the near-horizon geometry of a single $NS5$ -brane, where the resolution is that a single fivebrane does not develop a linear dilaton throat. It is natural to ask whether this is also the case for the intersection of a single fivebrane ($k_1 = 1$) with k_2 orthogonal ones.

One way to address this questions is the following. T dualizing in a direction transverse to the k_2 fivebranes (and thus along the single fivebrane orthogonal to them) gives rise to a single $NS5$ -brane in IIA string theory, which wraps an A_{k_2-1} ALE surface described by the equation $z_1^{k_2} + z_2^2 + z_3^2 = 0$ in C^3 . This system is believed [18] to have a linear dilaton throat of the form

$$\mathbb{R}^{1,1} \times \mathbb{R}_\phi \times S^1 \times \mathcal{M}_{k_2}/\Gamma, \tag{6.3}$$

where \mathcal{M}_{k_2} is the A-series $N = 2$ minimal model with $c = 3 - 6/k_2$ and Γ a discrete group.

The geometry (6.3) has very different properties from the throats discussed in section 4. The Poincare symmetry is not enhanced beyond $ISO(1,1)$ in this case. This is natural from

the intersecting fivebrane perspective since, as explained in section 4, the 2+1 dimensional nature of the system for $k_1, k_2 > 1$ relied on the existence of both throats.

The background (6.3) has some additional properties which are puzzling from the point of view of the fivebrane configuration. The near-horizon geometry of the intersecting fivebranes is expected to have an $SO(4) \times SO(4)$ global symmetry. The background (6.3) does not have such a symmetry. Furthermore, out of the eight supercharges that one expects from the brane perspective, only four are realized linearly.¹¹

We see that there is some tension between the properties of the throat geometry (6.3) and those expected from the near-horizon geometry of the intersecting fivebranes. It would be interesting to understand the relation between the two better.

6.3 Type IIA

The intersecting $NS5$ -brane background (4.3) is an exact solution of the classical equations of motion of IIA string theory as well.¹² It is natural to ask how the results of this paper generalize to that case. Many aspects of the discussion carry over to the IIA case. In particular, it is still true that the near-horizon geometry for coincident fivebranes is (1.3), and that it exhibits an enhanced super-Poincare symmetry.

One can also repeat the calculations of section 4.2 and find the spectrum of massless modes living near the intersection of the branes for $M_W \gg M_s$. As we show in appendix A, the result is very similar to the IIB case. Instead of $k_1 k_2$ currents (3.11) associated with left-moving chiral fermions Ψ_n^a , one finds $k_1 k_2$ modes of the RR fields that correspond to products of left and right-moving real (Majorana) fermions, $\bar{\Psi}_a^n \Psi_n^a$. It is thus natural to conjecture that each intersection of two $NS5$ -branes carries a Majorana fermion.

Some other elements of the IIB discussion do not have an obvious analog in the IIA case. In particular, there is no D-brane picture which allows one to analyze the low energy dynamics associated with the intersection. One expects that the strong coupling region should be treated by lifting to eleven dimensions. It is natural to ask whether the full theory preserves the enhanced symmetry of the near-horizon background (1.3). This requires a better understanding of the fate of the fermions, and will be left for future work.

6.4 Fundamental strings and $AdS_3 \times \mathbb{R} \times S^3 \times S^3$

Adding fundamental strings stretched in the directions (01) to the intersecting $NS5$ -branes, and taking the near-horizon limit, one finds the geometry $AdS_3 \times \mathbb{R} \times S^3 \times S^3$, which can be thought of as a limit of $AdS_3 \times S^1 \times S^3 \times S^3$ with the radius of the S^1 going to infinity. This system was studied from the point of view of holography in [20][21]-[22], but it remains enigmatic. Some of our results are directly relevant to the study of this system. For example, the fact that when we take the near-horizon limit of the fivebranes, the fermions at the intersection decouple, implies that the same is true after adding the fundamental strings and taking their near-horizon limit. Also, the fact that before adding the strings

¹¹The background (6.3) is a special case of the non-critical superstring construction of [19], where the symmetry structure was analyzed.

¹²A quick way to see that is to note that the CHS geometry is an exact worldsheet CFT with $c = 6$, and can be used to construct IIA backgrounds as well.

the system has an enhanced Poincare symmetry should have important implications for the structure of the spacetime CFT after adding the strings. We will leave a more detailed discussion of this interesting system for future work.

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A. A CFT analysis of separated fivebranes

A.1 Conventions

Consider $N = 1$ SCFT on $SU(2)_k$. Denote the bosonic $SU(2)$ currents by $K^a(z)$, $a = 1, 2, 3$, and the fermions by χ^a . The superconformal generator is

$$G = Q \left(\frac{1}{\sqrt{2}} K^+ \chi^- + \frac{1}{\sqrt{2}} K^- \chi^+ + K^3 \chi^3 + \chi^+ \chi^- \chi^3 \right). \quad (\text{A.1})$$

The currents satisfy the OPE algebra

$$\begin{aligned} K^3(z)K^3(w) &\sim \frac{\frac{1}{2}(k-2)}{(z-w)^2}, \\ K^+(z)K^-(w) &\sim \frac{k-2}{(z-w)^2} + \frac{2K^3(w)}{z-w}, \\ K^3(z)K^\pm(w) &\sim \pm \frac{K^\pm(w)}{z-w}, \end{aligned} \quad (\text{A.2})$$

and the fermions χ^a satisfy

$$\begin{aligned} \chi^+(z)\chi^-(w) &\sim \frac{1}{z-w}, \\ \chi^3(z)\chi^3(w) &\sim \frac{1}{z-w}. \end{aligned} \quad (\text{A.3})$$

A similar set of conventions will be used for $SL(2)_k$. The bosonic currents will be denoted by $J^a(z)$, and the fermions by $\psi^a(z)$. They satisfy the OPE algebra

$$\begin{aligned} J^3(z)J^3(w) &\sim -\frac{\frac{1}{2}(k+2)}{(z-w)^2}, \\ J^+(z)J^-(w) &\sim \frac{k+2}{(z-w)^2} - \frac{2J^3(w)}{z-w}, \\ J^3(z)J^\pm(w) &\sim \pm \frac{J^\pm(w)}{z-w}, \end{aligned} \quad (\text{A.4})$$

and

$$\begin{aligned}\psi^+(z)\psi^-(w) &\sim \frac{1}{z-w}, \\ \psi^3(z)\psi^3(w) &\sim -\frac{1}{z-w}.\end{aligned}\tag{A.5}$$

The $N = 1$ supercurrent is

$$G \sim Q \left(\frac{1}{\sqrt{2}} J^+ \psi^- + \frac{1}{\sqrt{2}} J^- \psi^+ - J^3 \psi^3 - \psi^+ \psi^- \psi^3 \right).\tag{A.6}$$

A.2 Six dimensional background – fivebranes on a circle

k type IIB fivebranes on a circle are described by the background

$$\mathbb{R}^{5,1} \times \left(\frac{SL(2, \mathbb{R})}{U(1)} \times \frac{SU(2)}{U(1)} \right) / Z_k.\tag{A.7}$$

The background must have $(1, 1)$ supersymmetry in $5 + 1$ dimensions. A technically useful way to think about this system is the following. Consider the spacetime

$$\mathbb{R}^{5,1} \times SL(2)_k \times SU(2)_k.\tag{A.8}$$

This is a twelve dimensional background with signature $(10, 2)$. In order to get the physical background (A.7), one can gauge the null $U(1)$ super-Kac-Moody algebra

$$\frac{1}{\sqrt{2}}(\chi^3 - \psi^3) + \theta \frac{Q}{\sqrt{2}} \left(J_3^{(\text{tot})} - K_3^{(\text{tot})} \right),\tag{A.9}$$

where $J_3^{(\text{tot})} = J_3 + \psi^+ \psi^-$ is the total J_3 current, which receives contributions from bosons and fermions on the worldsheet, and similarly for $K_3^{(\text{tot})}$.

The gauging of the null superfield (A.9) implies the presence of a bosonic (β', γ') ghost system with spin $\frac{1}{2}$, and BRST charge

$$Q_{BRST} = \dots + \oint \frac{dz}{2\pi i} \gamma'(z) \frac{1}{\sqrt{2}} (\chi^3 - \psi^3).\tag{A.10}$$

As usual, we can write

$$\gamma' = \eta' e^{\varphi'},\tag{A.11}$$

where φ' is a canonically normalized scalar field with no linear dilaton.

We can bosonize the free fermions corresponding to (A.8) as follows:

$$\begin{aligned}\psi^\mu &\rightarrow H_1, H_2, H_3, \\ \chi^+ \chi^- &= i\partial H_4, \\ \psi^+ \psi^- &= i\partial H_5, \\ \frac{1}{\sqrt{2}}(\chi^3 - \psi^3) &= e^{iH'}.\end{aligned}\tag{A.12}$$

such that

$$Q_{BRST} = \dots + \oint \frac{dz}{2\pi i} \eta' e^{\varphi' + iH'} . \tag{A.13}$$

The spacetime supercharges take the form

$$Q_{\alpha}^{\pm} = \oint \frac{dz}{2\pi i} e^{-\frac{\varphi'}{2} - \frac{i}{2}H'} e^{-\frac{\varphi}{2}} S_{\alpha} e^{\pm \frac{i}{2}(H_4 + H_5)} . \tag{A.14}$$

Here S_{α} is a spinor of $\text{Spin}(5, 1)$ with a particular chirality, *i.e.* $\alpha \in \mathbf{4}$. It is instructive to check that (A.14) is indeed BRST invariant, w.r.t. the BRST charge of the $N = 1$ string.

The supercharges Q_{α}^{\pm} satisfy the conjugation relation $(Q^+)^\dagger = Q^-$, and the anti-commutation relation $\{Q_{\alpha}^+, Q_{\beta}^-\} = \gamma_{\alpha\beta}^{\mu} P_{\mu}$. To prove the latter, one uses the fact that $e^{-\varphi' - iH'}$ is a picture-changed version of the identity, as can be checked by applying $\{Q_{BRST}, \xi'\}$ to it.

In the type IIB theory, the other worldsheet chirality gives rise to supercharges $\bar{Q}_{\dot{\alpha}}$, which have the opposite chirality under $SO(5, 1)$,

$$\bar{Q}_{\dot{\alpha}}^{\pm} = \oint \frac{d\bar{z}}{2\pi i} e^{-\frac{\bar{\varphi}'}{2} - \frac{i}{2}\bar{H}'} e^{-\frac{\bar{\varphi}}{2}} \bar{S}_{\dot{\alpha}} e^{\pm \frac{i}{2}(\bar{H}_4 + \bar{H}_5)} . \tag{A.15}$$

In the IIA case \bar{Q} has the same chirality as Q .

We next discuss some observables and states in the theory from the present perspective. Consider first the (NS,NS) sector. A class of non-normalizable vertex operators which correspond to symmetric traceless operators in the low energy gauge theory is given by

$$\text{Tr} \Phi^{i_1} \Phi^{i_2} \dots \Phi^{i_{2j+2}}(p_{\mu}) \leftrightarrow e^{-\varphi - \bar{\varphi}} \left(\chi \bar{\chi} V_j^{(\text{su})} \right)_{j+1; m, \bar{m}} V_{j'; m, \bar{m}}^{(\text{sl})} e^{ip \cdot x} . \tag{A.16}$$

Here Φ^i are scalars in the adjoint of $SU(k)$. The operators on the l.h.s. of (A.16) are symmetric and traceless in the indices $(i_1, i_2, \dots, i_{2j+2})$. On the r.h.s., the indices (m, \bar{m}) are the same for $SU(2)$ and $SL(2)$, due to the constraint that (A.9) should vanish. The $SL(2)$ quantum number j' is determined via the mass-shell condition

$$\frac{j(j+1)}{k} - \frac{j'(j'+1)}{k} + \frac{1}{2}p^2 = 0 . \tag{A.17}$$

Note that the $SO(4)$ symmetry corresponding to rotations of the space transverse to the fivebranes is realized on the r.h.s. as the $SU(2)_L \times SU(2)_R$ symmetry generated by $K_a^{(\text{tot})}$, $\bar{K}_a^{(\text{tot})}$. The gauging of (A.9) breaks this symmetry down to $U(1) \times Z_k$. This is in agreement with the spacetime picture, where the breaking is due to the expectation value of one of the complex scalars Φ^i , which we will denote by B [13],[14].

We are particularly interested in the spectrum of on-shell massless states. These correspond to principal discrete series states in $SL(2, \mathbb{R})$, with $m = \bar{m} = j' + 1$ in (A.16). The masslessness condition implies, via (A.17) that $j' = j$. Hence we have normalizable states of the form (A.16) with $m = \bar{m} = j + 1 = j' + 1$, or more explicitly,

$$e^{-\varphi - \bar{\varphi}} \chi^+ \bar{\chi}^+ V_{j; j, j}^{(\text{su})} V_{j; j+1, j+1}^{(\text{sl})} e^{ip \cdot x} \tag{A.18}$$

with $p^2 = 0$. These correspond in the low energy gauge theory to the states

$$\frac{1}{2j+2} \text{Tr} B^{2j+2} |0\rangle = \sum_{l=1}^k b_l e^{2\pi i \frac{l(2j+1)}{k}} |0\rangle, \quad (\text{A.19})$$

with b_l the eigenvalues of B and $2j+1 = 1, 2, 3, \dots, k-1$. Thus, we see here the $k-1$ massless fields corresponding to the eigenvalues of B in the adjoint of $SU(k)$.

We next move on to the (R,R) sector, in particular the operators dual to $\text{Tr} F_{\mu\nu} B^{2j+1}$. The normalizable states corresponding to these operators have the form

$$e^{-\frac{\varphi}{2} - \frac{\bar{\varphi}}{2} - \frac{\varphi'}{2} - \frac{i}{2} H' - \frac{\bar{\varphi}'}{2} - \frac{i}{2} \bar{H}'} \xi_{\mu\nu} \gamma_{\dot{\alpha}\alpha}^{\mu\nu} S_{\dot{\alpha}} \bar{S}_{\alpha} e^{ip \cdot x} e^{\frac{i}{2}(H_4 + \bar{H}_4)} V_{j;j,j}^{(\text{su})} e^{-\frac{i}{2}(H_5 + \bar{H}_5)} V_{j;j+1,j+1}^{(\text{sl})}. \quad (\text{A.20})$$

Note that this satisfies $J_3^{(\text{tot})} = K_3^{(\text{tot})}$ and $\bar{J}_3^{(\text{tot})} = \bar{K}_3^{(\text{tot})}$, as necessary for (A.9) (of course, the same is true for (A.18)). (A.20) are normalizable vertex operators corresponding to the $U(1)^{k-1}$ photons

$$\sum_{l=1}^k F_{\mu\nu}^{(l)} e^{2\pi i \frac{l(2j+1)}{k}}. \quad (\text{A.21})$$

Again, as in (A.19), the overall $U(1)$ degree of freedom is absent. The states (A.20), (A.21) were discussed in [15]; the description here gives a particularly simple construction of these states.

A.3 Two dimensional background — intersecting fivebranes

We next move on to the case of two sets of fivebranes in IIB string theory:

- (1) k_1 NS5-branes stretched in (012345).
- (2) k_2 NS5-branes stretched in (016789).

We will spread the fivebranes in each set on a circle, as before, to regularize the problem. The background of interest will now be

$$\mathbb{R}^{1,1} \times SL(2)_{k_1} \times SU(2)_{k_1} \times SL(2)_{k_2} \times SU(2)_{k_2}. \quad (\text{A.22})$$

This is a fourteen dimensional spacetime of signature $11+3$. To get to the ten dimensional fivebrane geometry, we need to gauge two null $U(1)$'s of the form (A.9), one for each $SL(2) \times SU(2)$ factor in (A.22).

For brevity, we are going to omit below factors analogous to $e^{-\frac{1}{2}(\varphi' + iH')}$ in (A.14), etc. As we saw in the previous subsection, these factors play no role in the discussion since $e^{-r(\varphi' + iH')}$ has dimension zero for all r , and if $r \in Z_+$, the operator is proportional to the identity operator (after picture changing).

We will use the following notation for the free fermions:

$$\begin{aligned} \psi_0 \psi_1 &= i\partial H, \\ \chi_+^{(1)} \chi_-^{(1)} &= i\partial H_1, \\ \psi_+^{(1)} \psi_-^{(1)} &= i\partial H_2, \\ \chi_+^{(2)} \chi_-^{(2)} &= i\partial H_3, \\ \psi_+^{(2)} \psi_-^{(2)} &= i\partial H_4. \end{aligned} \quad (\text{A.23})$$

As we have seen, the SUSY corresponding to this fivebrane configuration in type IIB is four supercharges from the left and four from the right, all with the same chirality in (01). These supercharges can be constructed in analogy to (A.14), (A.15):

$$\begin{aligned} Q^{\pm,\pm} &= \oint e^{-\frac{\varphi}{2} + \frac{i}{2} H \pm \frac{i}{2} (H_1 + H_2) \pm \frac{i}{2} (H_3 + H_4)} , \\ \overline{Q}^{\pm,\pm} &= \oint e^{-\frac{\overline{\varphi}}{2} + \frac{i}{2} \overline{H} \pm \frac{i}{2} (\overline{H}_1 + \overline{H}_2) \pm \frac{i}{2} (\overline{H}_3 + \overline{H}_4)} . \end{aligned} \quad (\text{A.24})$$

Let us start with the question of what normalizable massless states there are in this geometry. We need to consider principal discrete series states in each $SL(2)$. It is not difficult to see that no analogs of the (NS,NS) states (A.18) can survive the GSO projection. The only states that can possibly make it are (RR) ones. They are described by the vertex operators (compare to (A.20))

$$\begin{aligned} &e^{-\frac{\varphi}{2} - \frac{\overline{\varphi}}{2}} e^{\frac{i}{2} (H + \overline{H})} e^{ip \cdot x} e^{\frac{i}{2} (H_1 + \overline{H}_1)} V_{j_1; j_1, j_1}^{(su_1)} e^{-\frac{i}{2} (H_2 + \overline{H}_2)} V_{j_1; j_1+1, j_1+1}^{(sl_1)} \\ &e^{\frac{i}{2} (H_3 + \overline{H}_3)} V_{j_2; j_2, j_2}^{(su_2)} e^{-\frac{i}{2} (H_4 + \overline{H}_4)} V_{j_2; j_2+1, j_2+1}^{(sl_2)} . \end{aligned} \quad (\text{A.25})$$

For each (j_1, j_2) with $2j_i = 0, 1, 2, \dots, k_i - 2$, we have a chiral scalar. In the convention where the supercharges satisfy $Q^2 \sim P^+$, the states (A.25) have $P^+ = 0$, $P^- \neq 0$. The total number of chiral scalars is $(k_1 - 1)(k_2 - 1)$. Looking at the transformation properties under $SO(4)_{2345} \times SO(4)_{6789}$, we see that these scalars carry quantum numbers of

$$\text{Tr} \Psi B_1^{2j_1+1} \Psi^* B_2^{2j_2+1} . \quad (\text{A.26})$$

They correspond to the currents (3.15).

A useful interpretation of the $(k_1 - 1)(k_2 - 1)$ holomorphic currents (A.25) is as the RR ground states of the CFT on the product of cigars and minimal models. Each minimal model has $k_i - 1$ chiral operators, which are mapped by spectral flow to $k_i - 1$ RR ground states. At a generic point in the moduli space of fivebrane positions, all these are normalizable, and we have $(k_1 - 1)(k_2 - 1)$ RR ground states.

It is easy to generalize the above analysis to the case of $NS5$ -branes intersecting in IIA string theory. In the expression for the supercharges (A.24), and RR vertex operators (A.25), one simply takes $H \rightarrow H$, $\overline{H} \rightarrow -\overline{H}$. This leads to a superalgebra with four left-moving and four right-moving (in spacetime) supercharges, and the RR states (A.25) have the quantum numbers of

$$\text{Tr} \Psi B_1^{2j_1+1} \overline{\Psi} B_2^{2j_2+1} , \quad (\text{A.27})$$

where Ψ_n^a are real left-moving (Majorana Weyl) fermions, and $\overline{\Psi}_a^n$ are their right-moving counterparts.

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